

Gravity operates by the inverse square law

(source Hyperphysics)

A main objective in this lesson is that you understand the basic notion of “inverse square” relationships. There are a small number (perhaps less than 25) general paradigms of nature that if you make them part of your basic view of nature they will help you greatly in your understanding of how nature operates.

Gravity is the weakest of the [four fundamental forces](#), yet it is the dominant force in the universe for shaping the large-scale structure of galaxies, stars, etc. The gravitational force between two masses m_1 and m_2 is given by the relationship:

$$F_{\text{gravity}} = \frac{Gm_1 m_2}{r^2} \quad \text{where} \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

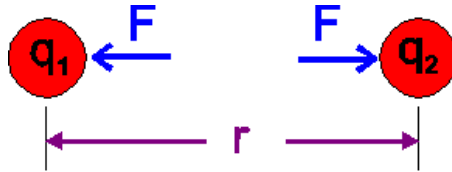
This is often called the "universal law of gravitation" and G the universal gravitation constant. It is an example of an [inverse square law](#) force. The force is always attractive and acts along the line joining the centers of mass of the two masses. The forces on the two masses are equal in size but opposite in direction, obeying [Newton's third law](#). You should notice that the universal gravitational constant is REALLY small so gravity is considered a very weak force.

The gravity force has the same form as [Coulomb's law](#) for the forces between electric charges, i.e., it is an inverse square law force which depends upon the product of the two interacting sources. This led Einstein to start with the [electromagnetic force](#) and gravity as the [first attempt](#) to demonstrate the [unification](#) of the fundamental forces. It turns out that this was the wrong place to start, and that gravity will be the last of the forces to unify with the other three forces. [Electroweak unification](#) (unification of the electromagnetic and weak forces) was demonstrated in 1983, a result which could not be anticipated in the time of Einstein's search. It now appears that the common form of the gravity and electromagnetic forces arises from the fact that each of them involves an exchange particle of zero mass, not because of an inherent symmetry, which would make them easy to unify.

Coulomb's Law operates by the inverse square law too

Any two charged particles will exert a force on each other. Opposite charges will produce an attractive force while similar charges will produce a repulsive force. The greater the charges, the greater the force. The greater the distance between them, the smaller the force. For two points separated by a distance r the formula is:

$$F = \frac{kq_1q_2}{r^2}$$



- F the force on each charge, + indicates repulsion, - indicates attraction
- k is a constant—a big number equal to 9×10^9 Newton.meters/Coulombs²
- q_1 the quantity of charge 1 measured in coulombs
- q_2 the quantity of charge 2 measured in coulombs
- r the radius of separation from center of one charge to the center of the other.

You should notice that the formula for Coulomb's law and the Gravitational Law are very similar in that they both operate by the inverse square and both have constants. THEY ARE different in that the constant G is very small and the constant k is very big. They also differ in the gravity is always attractive and charges can attract or repel. One uses the product of the charges, the other uses the product of the masses.

$$F = \frac{kq_1q_2}{r^2} \quad F_G = \frac{Gm_1m_2}{r^2}$$

The ratio of the electrical force to that of the gravitational force turns out to be 10^{42} . This is a colossal number! Suppose we are studying a physics problem involving the motion of particles in a box under the action of two forces with the same range, but differing in magnitude by a factor 10^{42} . It would seem a plausible approximation (to say the least) to start the investigation by neglecting the weaker force. Applying this reasoning to the motion of particles in the Universe, we would expect the Universe to be governed entirely by electrical forces. This we find is not the case. The force

that holds us to the surface of the Earth, and prevents us from floating off into space, is gravity. The force which causes the Earth to orbit the Sun is also gravity. In fact, on astronomical length-scales gravity is the dominant force, and electrical forces are largely irrelevant. The key to understanding this paradox is that there are both positive and negative electric charges, whereas there are only positive gravitational "charges." This means that gravitational forces are always cumulative, whereas electrical forces can cancel one another out. Suppose, for the sake of argument, that the Universe starts out with randomly distributed electric charges. Initially, we expect electrical forces to completely dominate gravity. These forces try to make every positive charge get as far away as possible from the other positive charges, and as close as possible to the other negative charges. After a while, we expect the positive and negative charges to form close pairs. Just how close is determined by quantum mechanics, but, in general, it is pretty close: *i.e.*, about 10^{10} m. The electrical forces due to the charges in each pair effectively cancel one another out on length-scales much larger than the mutual spacing of the pair. It is only possible for gravity to be the dominant long-range force if the number of positive charges in the Universe is almost equal to the number of negative charges. In this situation, every positive charge can find a negative charge to team up with, and there are virtually no charges left over. In order for the cancellation of long-range electrical forces to be effective, the relative difference in the number of positive and negative charges in the Universe must be incredibly small. In fact, positive and negative charges have to cancel each other out to such accuracy that most physicists believe that the net charge of the universe is *exactly* zero. But, it is not enough for the Universe to start out with zero charge. Suppose there were some elementary particle process which did not conserve electric charge. Even if this were to go on at a very low rate, it would not take long before the fine balance between positive and negative charges in the Universe was wrecked. So, it is important that electric charge is a *conserved* quantity (*i.e.*, the net charge of the Universe can neither increase or decrease). As far as we know, this is the case. To date, no elementary particle reactions have been discovered which create or destroy net electric charge.

(<http://farside.ph.utexas.edu/teaching/em/lectures/node28.html>)

For a video explaining Coulomb's law →

<http://www.youtube.com/watch?v=rYjo774UpHI>

Charles Augustin de Coulomb



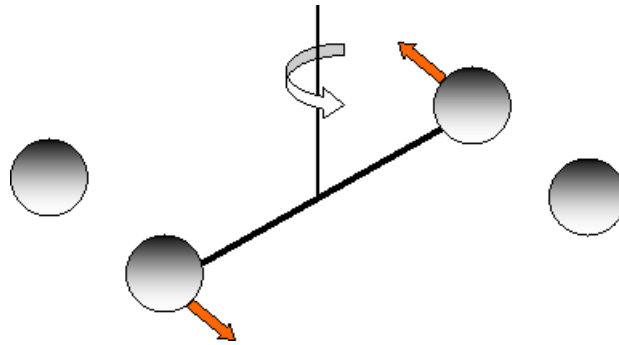
Source <http://www.studyphysics.ca/30/coulomb.pdf>

A little about Coulomb.

- He was born in 1736 in Angoulême, France.
- He received the majority of his higher education at the Ecole du Genie at Mezieres (sort of the French equivalent of universities like Oxford, Harvard, etc.) from which he graduated in 1761.
- He then spent some time serving as a military engineer in the West Indies and other French outposts, until 1781 when he was permanently stationed in Paris and was able to devote more time to scientific research.

Between 1785-91 he published seven memoirs (papers) on physics.

- One of them, published in 1785, discussed the inverse square law of forces between two charged particles. This just means that as you move charges apart, the force between them starts to decrease faster and faster (exponentially).
- In a later memoir he showed that the force is also proportional to the product of the charges, a relationship now called “Coulomb’s **Law**”.
- For his work, the unit of electrical charge is named after him. This is interesting in that Coulomb was one of the first people to start creating the metric system.
- He died in 1806. **The Torsion Balance** When Coulomb was doing his original experiments he decided to use a torsion balance to measure the forces between charges.
- Henry Cavendish’s experiment to measure the value of “G” , the universal gravitational constant.
- Coulomb was actually doing his experiments about 10 years before Cavendish.
- He set up his apparatus as shown below with all spheres charged the same way.



- He charged one of the free moving spheres.
- He then touched it to the other free moving sphere (charging it by conduction).
- Each of the free moving spheres was then touched to one of the spheres on the rod.
- Although he didn't know the actual charge on any particular sphere, Coulomb did know that each was equal.
- Coulomb also altered the experiment by using spheres of different sizes so he could get the amounts of charge in different ratios.

Because like charges repel, the spheres on the torsion balance twist away from the other balls. • By knowing the distance between the balls, the force needed to twist them (the torque from

which the torsion balance gets its name), and the charges on the balls, he could figure out a formula.

After all this work Coulomb finally came up with a formula that could be used to calculate the force between any two charges separated by a distance...

This all was started by Newton when he began trying to figure out the value for the universal gravitational constant G . Newton started with the idea that since the Earth is pulling on the apple, the apple must also be pulling on the Earth (Newton's 3rd Law).

- Ask a person on the street where gravity comes from in this situation, and they'd probably say "the Earth". They would never consider the apple as a "source" of gravity.
- But if the apple is pulling on the Earth, that must mean that an object doesn't have to be huge to have a gravitational pull on other objects. There is nothing special about the Earth compared to the apple... **both** are sources of gravity.
- That would mean that one apple should be able to have a gravitational pull on another apple... that means **any mass** pulls on **any other mass**.
- The reason we don't see the effect of, for example, you being pulled towards your computer mouse, is that the masses are so small that the force is also very small.
- Still, the force is there, and Newton wanted to come up with a way of calculating it.

Using a lot of calculus and some pretty tough physics he came up with this formula:

$$F_g = \frac{Gm_1m_2}{r^2}$$

F_g = force due to gravity between the two objects (N)

G = the Gravitational Constant

m_1 and m_2 = the two masses (kg)

r = the distance between the two objects' centres (m)

This formula shows that any objects with mass will pull towards each other with a gravitational force.

- We usually say that an object has a **gravitational field** around it.
- Any **field** is just a sphere of influence around the object. The closer you get, the more you are affected by it.
- In the case of gravity, the bigger the mass of the object, the bigger the **field**.

- The formula also shows that the closer the objects are, the greater the effect of the gravitational field.

Newton then turned his attention to trying to find the value for the Gravitational Constant, “G”.

- Nope, it *isn't* the acceleration due to gravity on Earth, 9.81m/s².
- Newton looked for a way of calculating the value for G from the formula above.

If we solve that formula for G we get:

$$G = \frac{F_g r^2}{m_1 m_2}$$

Let's look at how we will substitute numbers into this formula.

- Newton realized that the only thing he could measure a F_g for would be an object on Earth's surface. An example would be you. We could calculate the force due to gravity on your body easily using $F_g = mg$.
- We need to know the distance from the centre of the Earth to your centre... which we do have: 6.38e6 m. And yup, they even had a pretty good estimate of this in Newton's time!
- We need to know your mass, which would be m_1 ... that's no problem.
- The last thing we need, m_2 , is the mass of the Earth. Oh, oh. That one is a problem.

In Newton's time no one had any idea how heavy the Earth really was.

If we knew G, then we could calculate the mass of the Earth, but that's what we are trying to calculate here!

- Newton continued to look for some way to calculate G indirectly, but never found a way.

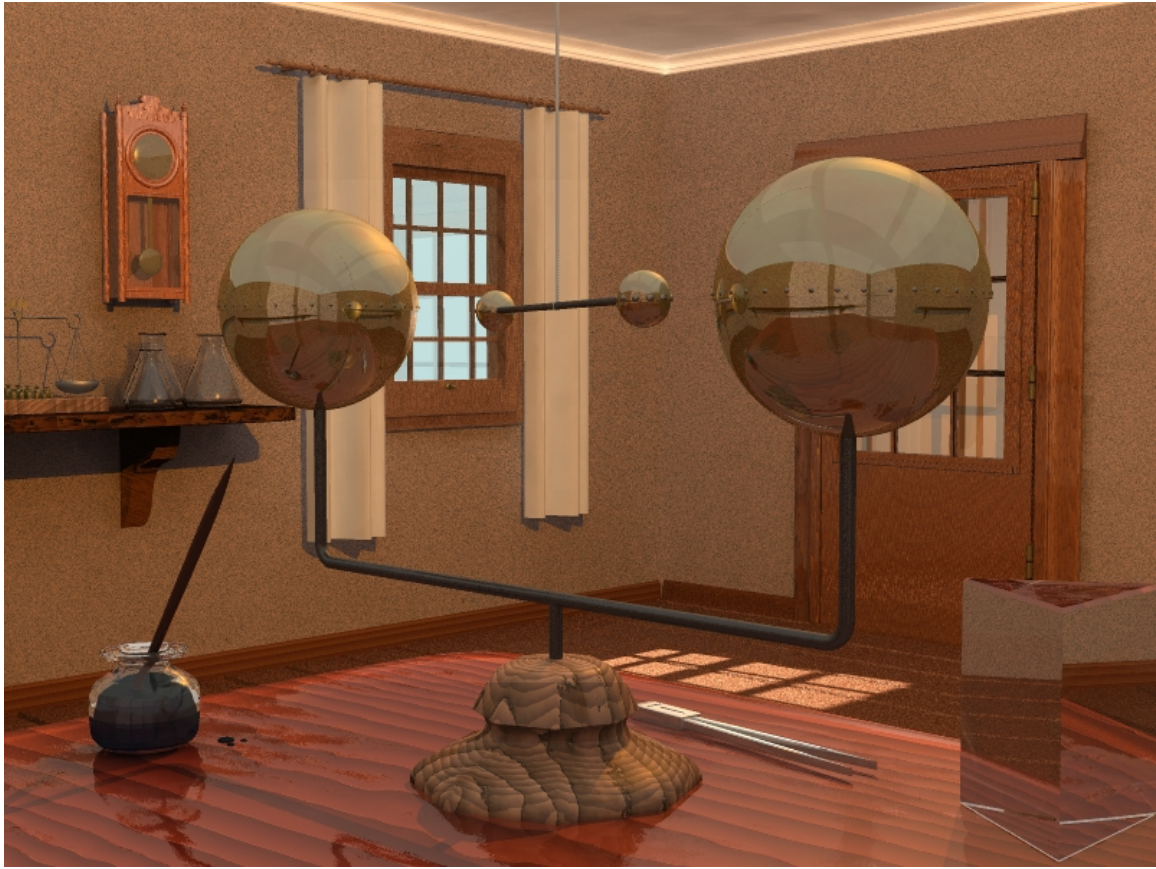
Henry Cavendish

About a 100 years later a man named Henry Cavendish finally figured out a way to measure the value for G. He used a torsion balance, which is an amazing invention. The twisting of a wire is the central component. A twisting force is called torque and torsion is a word that describes anything that is being twisted.

We do not have pictures of the Cavendish torsion balance since permanent picture photography was not invented until about 40 years later. Here are some images of what his and modern versions look like.



note the stool!



a very modern one



Cavendish's Torsion Balance

- He attached a really heavy pair of metal balls to the ends of a long metal rod, and then hung the rod from a wire.
- He then brought another pair of really heavy metal balls near the balls on the rod.
- Cavendish knew that because they had mass they should pull on each other, but very weakly.
- To measure this weak pull, he carefully measured how much the wire was twisting (torque) whenever he brought the other masses near by.
- This is why the device he used is called a **torsion balance**.

After a lot of very careful, very tedious tries, he found that G was

$$6.67^{-11} \text{Nm}^2/\text{kg}^2$$

- Cavendish realized that because he knew the value for G, he could now calculate the mass of the Earth. That's why he titled the paper that he published "**Weighing the Earth.**"

Example 1: Using values that you now know, **determine** the mass of the Earth.

We know that the force exerted on my body by the Earth is $F_g = mg$, where little “m” is my mass.

I also know that the force could be found using Newton's big Universal Gravitation Formula, where one mass is a little “m” (my mass), and the other mass is a big “ M_e ” (the mass of the Earth).

$$F_g = F_g$$

$$mg = \frac{GmM_e}{r^2}$$

$$g = \frac{GM_e}{r^2}$$

$$M_e = \frac{gr^2}{G}$$

$$M_e = \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$M_e = 5.99 \times 10^{24} \text{ kg}$$

Notice that we were able to combine a couple of formulas to get the new formula .

$$g = \frac{GM_e}{r^2}$$

- M_e does not always have to be the mass of the Earth. It could be the mass of the moon, Mars, an asteroid, whatever!